Evaluation of Pebble-Wise Integral Values Using Pebble Tracking Transport in GRIFFIN

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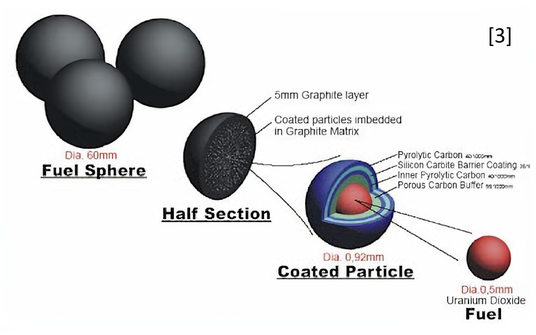
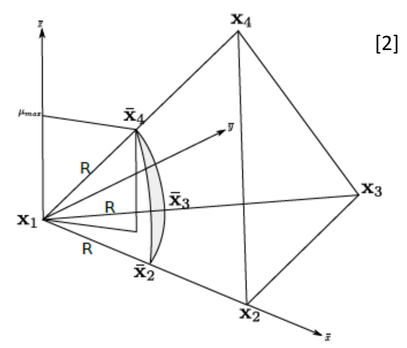
Introduction:

The Pebble Tracking Algorithm (PTT) is a method which enables GRIFFIN to perform high-fidelity discontinuous finite element (DFEM) neutron transport simulations for Pebble Bed Reactors (PBRs)[1]. PTT generates an unstructured tetrahedral mesh where each element vertex is located at a pebble center. Each tetrahedral element in the pebble region is therefore partially composed of at least one solid angle “slice” from a pebble[2].





DFEM is solved using the weak-form neutron transport equation represented by following equation.

$$\sum_{m=1}^{M} \omega_m \left[\left(\psi_m^*, -\vec{\Omega}_m \cdot \psi_m + \Sigma_t \right)_{\mathcal{D}} - \langle [\![\psi_m^*]\!], \psi_m^- \rangle_{\Gamma_{int}} + \langle \psi_m^*, \psi_m \rangle_{\partial D^+} \right] - \sum_{m=1}^{M} \omega_m (\psi_m^*, \Sigma_s \Phi)_{\mathcal{D}} = \sum_{m=1}^{M} \omega_m (\psi_m^*, S)_{\mathcal{D}}$$

Here, the inner products may be evaluated over an element using the following equation.

$$(\psi_m^*, \Sigma_t)_E = \sum_{i=1}^{N} \psi_{m,i}^* \sum_{j=1}^{N} \psi_{m,j} \int_E \Sigma_{t,g} b_i(\boldsymbol{x}) b_j(\boldsymbol{x}) d\boldsymbol{x}$$

where $b_i(\boldsymbol{x})$ are the shape functions, $\psi_{m,i}^*$ and $\psi_{m,j}$ are the expansion coefficients for the test functions and solution, and N is the total number of degrees of freedom (DoF) [2].

Evaluation of Integral Values:

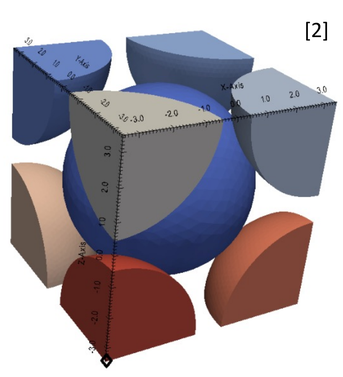
Cutting pebble regions with tetrahedral elements presents a unique challenge when evaluating pebble-wise integral values like reaction rates which are needed to evaluate the precision of PTT. One cannot simply sum integrals over entire elements. Our solution is to create an AuxKernel that loops over nodes, k, then loops over connected elements, e, from which a partial integral is evaluated with respect to the node of interest. The partial integral for pebble power is evaluated using the mass matrix, M as shown below.

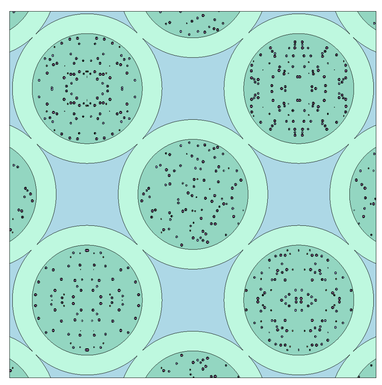
$$P_k = \sum_e^E \psi_{m,e}^{*T} \left[\left(\kappa \Sigma_f \right)_k M_{e,k} \right] \psi_{m,e}$$

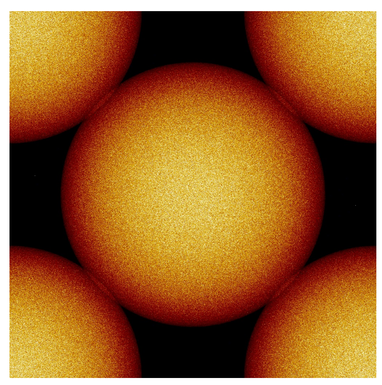
$$M_{e,k,i,j} = \sum_{l,m,n} \alpha_{l,m,n} \frac{R^{l+m+n+3}}{l+m+n+3} \int_0^{\mu_{max}} (1-\mu^2)^{l+m/2} \mu^y \left(\int_{\omega_{min}(\mu)}^{\omega_{max}(\mu)} cos^l(\omega) sin^m(\omega)\, d\omega \right) d\mu$$

Pebble-Wise Power in 9-Pebble Case:

The PTT algorithm was tested against SERPENT, a Monte-Carlo radiation transport code with the ability to calculate pebble powers. For SERPENT, the TRISO particles were individually modelled in each pebble, while homogenized macroscopic cross-sections were supplied by SERPENT for modelling in GRIFFIN. A 9-pebble body-centered cubic (BCC) lattice was modelled due to its simplicity. The corner pebbles were cut into octants and reflective boundary conditions were set for all boundary faces. Both models were normalized to a power of 2,000 W with an 11-group structure and anisotropy order of l=7.

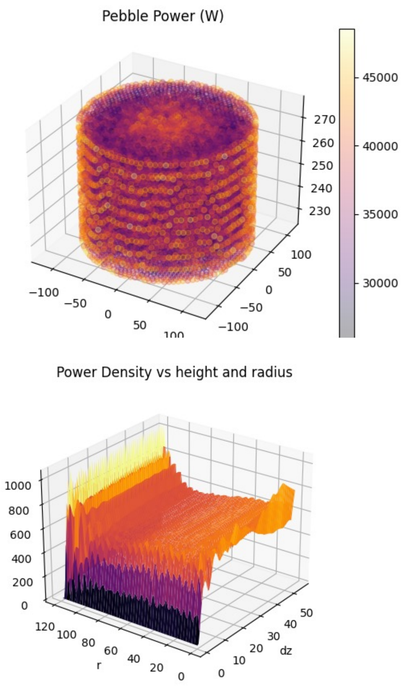






Note that the corner pebble values have different power values. This is due to having different distributions of TRISO particles. The homogenization of the cross sections in GRIFFIN ignore the spatial effects of the TRISO particles which is most likely the main source of error in the PTT simulation.

Pebble Center Coordinates			Power		Relative Error (%)
x (cm)	y (cm)	z (cm)	SERPENT	GRIFFIN	
3.46	3.46	3.46	124.89	121.29	2.888
3.46	3.46	-3.46	125.09	127.68	-2.069
3.46	-3.46	-3.46	125.04	125.84	-0.642
3.46	-3.46	3.46	124.93	114.15	8.624
0.00	0.00	0.00	1000.09	999.33	0.076
-3.46	3.46	3.46	125.05	129.44	-3.514
-3.46	3.46	-3.46	124.65	127.19	-2.036
-3.46	-3.46	-3.46	125.35	124.49	0.679
-3.46	-3.46	3.46	124.92	130.59	-4.538



Ongoing Work:

Our work aims to conduct deeper analysis of potential sources of error for the nine-pebble case, including testing with finer energy group structures and various scattering anisotropies. We will also extend the new capabilities in GRIFFIN to evaluate full-core PBRs and evaluate pebble-wise surface currents.

References:

1. Ref 1. –DINCER, I., & Dinçer İbrahim. (2018). *Comprehensive Energy Systems*. Elsevier.
2. Ref 2. –Wang, Y., Ortensi, J., Schunert, S., & Laboure, V. (2018). A Pebble Tracking Transport Algorithm for Pebble Bed Reactor Analysis. *OSTI*.
3. Ref 3. – Basak, U., & Nabliek, H. (n.d.). (rep.). *Performance Analysis Review of Thorium TRISO Coated Particles during Manufacture, Irradiation and Accident Condition Heating Tests*. Vienna, Austria: IAEA.